HEAT AND MASS TRANSFER WITH NATURAL CONVECTION AT A VERTICAL POROUS SURFACE, WITH BLOWING OF CARBON DIOXIDE INTO AIR

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This paper gives the results of a numerical calculation of a laminar boundary layer with the free convection of a binary mixture of carbon dioxide and air at a vertical heated surface. It compares the numerical solution with an approximate analytical solution and with experiment.

When several components are present in a boundary layer, the ordinary mechanism for the transfer of energy is complicated by diffusion effects. The differential equations of a laminar boundary layer, describing the free convection of a binary mixture along a vertical surface, taking account of the diffusional transfer of energy, have the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)+g\beta_m(m_1-m_{1\infty}) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\frac{c_{p1}-c_{p2}}{\rho_\infty c_{p\infty}}j_1\frac{\partial T}{\partial y}=-\frac{1}{\rho_\infty c_{p\infty}}\frac{\partial q}{\partial y} \tag{3}$$

$$u\frac{\partial m_1}{\partial x}+v\frac{\partial m_1}{\partial y}=-\frac{1}{\rho_\infty}\frac{\partial j_1}{\partial y} \tag{4}$$

$$q=-k\frac{\partial T}{\partial y}+\frac{\alpha_T RM^2T}{M_1M_2}j_1 \tag{5}$$

$$j_1=-\rho_\infty D\frac{\partial m_1}{\partial y} \tag{6}$$

The boundary conditions are

$$\begin{gathered} y=0,\quad u=0,\quad v=v_w,\quad T=T_w,\quad m_1=m_{1w}\\ y=\infty,\quad u=0,\quad T=T_\infty,\quad m_1=m_{1\infty}\end{gathered} \tag{7}$$

Here x is a coordinate directed along the surface; y is a coordinate directed perpendicular to the surface; u, v are the components of the velocity along the x and y axes; T, T_w, T_∞ are the temperatures, respectively, within the boundary layer, at the vertical surface, and beyond the limits of the boundary layer; m, m_w, m_∞ are the mass concentrations of the active component, respectively, within the boundary layer, at the vertical surface, and beyond the limits of the boundary layer; β_T, β_m are the temperature and concentration coefficients of volumetric expansion; ν is the kinematic viscosity; k is the coefficient of thermal conductivity; c_{p1} and c_{p2} are the specific heat capacities of the active component (carbon dioxide) and air; α_T is

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TABLE 1

f_w	$\theta'(0)$	$\varphi'(0)$	$f''(0)$	$N(\frac{1}{4}G)^{-1/4}$	m_{1w}	N/N_0
0	−0.5034	−0.0000	0.6454	0.5034	0	1
−0.005	−0.4842	−0.5986	0.6022	0.4716	0.027	0.957
−0.01	−0.4671	−0.5757	0.5561	0.4652	0.064	0.924
−0.02	−0.4302	−0.5236	0.4500	0.3834	0.151	0.762
−0.03	−0.3745	−0.4662	0.2338	0.3134	0.185	0.623

a′ thermodiffusional constant; M, M_1, M_2 are the molecular weights of the mixture of the active component and air; R is the gas constant of the mixture.

In the system of equations (1)-(4) the physical properties of the medium are assumed constant. The density of the medium, entering into the expression for the lifting force, depends on the temperature of the medium and on the concentration of the active component. The temperature of the surface T_W and the concentration of the active component at the surface, m_{1W}, are constant. The second term on the right side of Eq. (5) describes the transfer of heat by diffusional thermal conductivity (the Dufeau effect). Thermodiffusion was not taken into account in the determination of the flow of mass using Eq. (6).

We introduce the variable $\eta = c_1 y x^{-1/4}$, where

$$c_1 = \left[\frac{g\beta_T (T_w - T_\infty)}{4\nu^2}\right]^{1/4}$$

and the flow function

$$\psi = 4\nu c_1 x^{3/4} f(\eta) \tag{8}$$

which is such that

$$u = \frac{\partial\psi}{\partial x}, \quad v = -\frac{\partial\psi}{\partial y}$$

The system (1)-(4) is reduced to ordinary differential equations. In the new variables, the components of the velocity, u and v, have the form

$$u = 4\nu c_1^2 x^{1/2} f'(\eta), \quad v = \nu c_1 x^{-1/4} [f'(\eta) - 3f(\eta)] \tag{9}$$

and the equation of motion (2) is transformed:

$$f'''(\eta) + 3f(\eta) f''(\eta) - 2f'^2(\eta) + \theta(\eta) + e\varphi(\eta) = 0 \tag{10}$$

From the equations of energy (3) and diffusion (4), respectively, we obtain

$$\theta''(\eta) + [3Pf(\eta) + a\varphi'(\eta)]\theta'(\eta) - 3Scf(\eta)\varphi'(\eta) = 0 \tag{11}$$

$$\varphi''(\eta) + 3Sf(\eta)\varphi'(\eta) = 0 \tag{12}$$

where

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \varphi(\eta) = \frac{m_1 - m_{1\infty}}{m_{1w} - m_{1\infty}}, \quad e = \frac{\beta_m}{\beta_T}\frac{m_{1w} - m_{1\infty}}{T_w - T_\infty}$$

$$\beta_m = \frac{M_2 - M_1}{M_1 - (M_2 - M_1) m_{1w}}, \quad a = \frac{c_{p1} - c_{p2}}{c_p}(m_{1w} - m_{1\infty}) L$$

$$c = \frac{a_T R M^2 T_w}{c_p M_1 M_2}\frac{m_{1w} - m_{1\infty}}{T_w - T_\infty} L, \quad c_p = c_{p1} m_{1w} + c_{p2}(1 - m_{1w})$$

P is the Prandtl number; S is the Schmidt number; L is the Lewis number.

The boundary conditions in the new variables are

$$\begin{gathered} \eta = 0, \quad f' = 0, \quad f_w = \text{const}, \quad \theta = 1, \quad \varphi = 1 \\ \eta = \infty, \; f' = 0, \; \theta = 0, \; \varphi = 0 \end{gathered} \tag{13}$$

The value of f_W is determined from the condition of the semipermeability of the vertical surface (the surface is not permeable for air)

$$f_w = -\frac{1}{3S}\frac{m_{1\infty} - m_{1w}}{1 - m_{1w}}\varphi'(0) \tag{14}$$

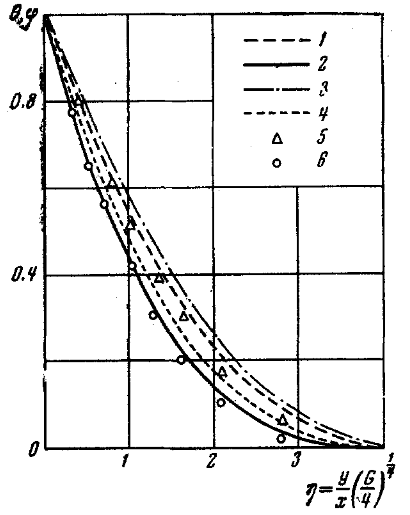

Fig. 1

The system of nonlinear differential equations (10)-(12) with the boundary conditions (13) was solved by the numerical step method on an M-20 computer, using an iteration process [1, 2].

The results of a calculation for a binary mixture of carbon dioxide and air are given in Table 1. For the calculation, the following quantities are given: T_W/T_∞, P, S (or L)

$$D_D = \frac{a_T RM^2 T_w}{M_1 M_2 C_p (T_w - T_\infty)}$$

where D_D is the Dufeau number. The parameters a, c, e are computed as a function of the blowing parameters. The values can be determined from the data of the table.

The heat flux in a binary mixture depends on the molecular and diffusional heat conductivity and is determined by formula (5), which is transformed to the form

$$q_x = -kcx^{-1/4}(T_w - T_\infty)\{\theta'(0) + LD_D(m_{1w} - m_{1\infty})\varphi'(0)\}$$

or, in dimensionless form

$$N_x = -(^1/_4 Gx)^{1/4}[\theta'(0) + c\varphi'(0)] \qquad (15)$$

where the coefficient C is determined above.

Analogously, from relationship (6), which corresponds to the diffusion of mass, without taking account of thermal diffusion, for the Sherwood number we find

$$N_D = -(^1/_4 Gx)^{1/4}\varphi'(0) \qquad (16)$$

As a result, we can plot the profiles of the temperature, the velocity, and the concentration in the boundary layer, as well as the flows of heat and mass.

Experiments were carried out on the determination of the concentration profiles with the blowing of carbon dioxide into air at a vertical porous surface in a laminar boundary layer, with the simultaneous transfer of heat and mass, under conditions of natural convection. The carbon dioxide was blown through a porous copper plate measuring 200×300 mm^2, which was encased in a hermetically sealed housing, having an independent heating system. The plates were heated using radiant heaters. This method of heating makes it possible to obtain a homogeneous temperature over the whole working surface.

The radiant heaters were made in the form of a double row of mirror-type infrared lamps. The degree of heating of the porous plate was monitored by the change of the voltage in the feed circuit of the lamps. The mass flow rate of the gas was measured with an RS-3 rotameter. Control over the uniformity of the heating and measurement of the temperature of the plate were effected by copper–constantan thermocouples, embedded in the surface of the plate.

The thermocouples were made of copper wire with a diameter of 0.1 mm and constantan wire with a diameter of 0.15 mm.

The readings of the thermocouples were recorded on an R-306 potentiometer. A Mach-Zehnder interferometer was used to determine the temperature and concentration fields. The light flux was directed parallel to the short side of the plate. The interferograms were analyzed using the formula

$$X_1 = \left[\frac{\lambda ST}{lp} + \frac{K_2}{R_2}\left(\frac{T}{T_\infty} - 1\right)\right]\left(\frac{K_1}{R_1} - \frac{K_2}{R_2}\right)^{-1} \qquad (17)$$

Here λ is the wavelength of the monochromatic light; S is the dimensionless shift of the interference band; X is the volumetric concentration; p is the pressure; K is the Dahl-Gladstone constant; l is the width of the model.

The mass concentration is connected with the volumetric concentration by the following relationship:

$$m_1 = M_1 X_1 [M_1 X_1 + M_2(1 - X_1)]^{-1}$$

To obtain the concentration profiles by analysis of the interferograms using formula (17), we need to know the temperature field in the cross section under investigation. The temperature in the boundary layer was measured using a positioner.

The thermocouple of the positioner was made of the same wires as the thermocouples for measuring the surface temperature of the plate. The readings were recorded in an electronic ribbon-type automatic recorder.

Figure 1 compares the temperature and concentration profiles, obtained by experimental and numerical methods, with the data of [3], which gives an approximate analytical solution. Curves 1, 3, and points 5 represent the numerical, approximate [3], and experimental temperature profiles in the boundary layer, with $f_W = -0.01$, $P = 0.71$, $S = 0.86$, $D_D = 0.2$, $L = 0.83$, and $T_W/T_\infty = 1.1$. Curves 2, 4, and points 6 represent the corresponding concentration profiles for carbon dioxide.

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